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TRANSMISSION FROM A FLAT PLATE TO
A FLUID BLOWING AT A HIGH VELOCITY

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TRANSMISSION OF HEAT FROM A FLAT PLATE TO
A FLUID FLOWING AT A HIGH VELOCITY*

By Luigi Crocco

SUMMARY

The writer, starting with the consideration of the hydrodynamic and thermodynamic equations for the turbulent boundary layer of a flat plate when it is necessary to take into account the heat produced by friction, arrives at the conclusion that the transmission of the heat follows the same law that is valid when the frictional heat is negligible, provided the temperature of the fluid is considered to be that which the fluid would reach if arrested adiabatically. It is then shown how the same law holds good for faired bodies, and some applications of the law are made to the problems of flight at very high speeds.

The ordinary theory of heat transmission between a flat plate and a flowing fluid completely ignores the effect of the heat developed by friction on the temperature near the plate and on the coefficient of transmission. In practice, the heat produced by friction may be disregarded so long as the velocity is kept within normal limits. At very high speeds, however, toward which modern flight is tending, the effect of friction is so great as to lead to entirely erroneous results even from the qualitative viewpoint, unless this effect is taken into account. Hence it is necessary to determine how the customary heat-transmission formulas can be adapted to this case. In the present article, which constitutes the essential part of a paper recently presented to the R. Accademia dei Lincei (reference 1), this result is accomplished in a remarkably simple manner, as follows.

In our case the hydrodynamic equations for the boundary layer are reduced to the following:

*"Sulla trasmissione del calore da una lamina piana a un fluido scorrente ad alta velocita." L'Aerotecnica, February, 1932, pp. 181-197.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left(v \frac{\partial u}{\partial y} \right) \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

For $y = 0$, $u = v = 0$. For $y = \delta$ (thickness of boundary layer), $u = U$ (velocity of undisturbed flow) and

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left(v \frac{\partial u}{\partial y} \right) = 0.$$

To this may be added the thermodynamic equation expressing the heat balance for a volumetric element of the fluid, an equation which, simplified by Pohlhausen (reference 2) for the present case, joins the temperature field to the velocity field:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left(\frac{\lambda}{\gamma c_p} \frac{\partial T}{\partial y} \right) + \frac{v}{g \beta c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (2)$$

The symbols have the usual meaning. Anyhow the symbol for the specific heat is written c_p , because the investigation is of practical importance only for gaseous fluids.

The effect of the frictional heat is included in the last term, whose quadratic dependence on the velocity field explains why it may be disregarded so long as it does not attain high values. Just for this reason, this investigation will be confined to the case of the turbulent boundary layer. It is possible, however, on the basis of Pohlhausen's results, to effect the required solution and to calculate the transmission by a method similar to the following, although more complicated.

The greater simplicity for turbulent flow is due to the ability in this case to write

$$\sigma = \gamma c_p \frac{v_{\text{turb}}}{\lambda} = 1, \quad \text{Prandtl's law}$$

while the analogous coefficient in laminar or parallel flow differs from unity, though constant for each fluid. By utilizing this relation in equation (2) and adding it to the first of equations (1) multiplied by $u/g\beta c_p$, we obtain the following equation;

$$u \frac{\partial}{\partial x} \left(T + \frac{u^2}{2g E c_p} \right) + v \frac{\partial}{\partial y} \left(T + \frac{u^2}{2g E c_p} \right) =$$

$$= \frac{\partial}{\partial y} \left[v_{\text{turb}} \frac{\partial}{\partial y} \left(T + \frac{u^2}{2g E c_p} \right) \right]; \quad (3)$$

of a form perfectly analogous to the first of equations (1) in u and therefore satisfied by putting

$$T + \frac{u^2}{2g E c_p} = a u + \text{const.}$$

This solution appears as a superposition of ~~the~~ ^{two others}. The first,

$$T_1 + \frac{u^2}{2g E c_p} = \text{const.}, \quad (1)$$

is a particular solution of equation (2), characterized by the fact that all the heat developed by friction remains in the boundary layer. (Reference 3.) ~~Since~~ it is easily recognized by noting that the gradient of T , to which the flow of heat is proportional, ~~is~~ $u \partial u / \partial y$ and is therefore zero for $y = 0$ and for $y = \delta$. This solution is therefore valid when the surface of the plate is impermeable to heat (*insulated*).

The other solution, $T_2 = a u + \text{const.}$, is, instead, a solution of the homogeneous equation corresponding to equation (2) and obtained from the latter by eliminating the last term. Hence it does not take account of the frictional heat and represents the temperature field due to the flow of heat from the wall into an ideal fluid or, approximately, into an actual fluid flowing at a low velocity. In such a case and for a turbulent flow, it appears that the two fields of temperature and velocity are similar, as demonstrated by Prandtl and confirmed by experiments.* Under these conditions the calculation of the heat transmission from the wall is immediate. (2)

Indicating by T_{2p} and $T_{2\delta}$ the respective values of T_2 at the wall and on the surface of the boundary layer, the similitude can be expressed in the following form

$$T_2 - T_{2\delta} = (T_{2p} - T_{2\delta}) \left(1 - \frac{u}{U} \right) \quad (4)$$

*See account of the experiments of Franz Elias in Abhandlungen aus dem Aerodyn. Inst. an der Tech. Hochschule Aachen, No. 9.

Now the quantity of heat taken from a surface of unit width and length l , all being retained in the boundary layer, is represented by

$$Q = \int_0^{\delta} \gamma u c_p (T_2 - T_{2\delta}) dy \quad (5)$$

By substituting the preceding expression for $T_2 - T_{2\delta}$, we obtain

$$Q = (T_{2p} - T_{2\delta}) \int_0^{\delta} \gamma c_p u \left(1 - \frac{u}{U}\right) dy;$$

so that the heat carried off is easily calculated when a logical distribution is assumed for u .

Latzko (reference 4), by utilizing the well-known law $u = U \left(\frac{y}{\delta}\right)^{1/7}$ deduced by Karman from the empirical formula of Blasius for the resistance of tubes, found that

$$\frac{Q}{l} = 0.0356 \gamma c_p U \left(\frac{v}{U l}\right)^{1/5} (T_{2p} - T_{2\delta}).$$

Experience has shown that, with increasing Reynolds Numbers, the exponent of the law of velocity distribution decreases and may raise doubts as to the validity of Latzko's formula at high velocities. On the contrary, one is easily convinced when he reflects on the intimate connection between the coefficient of heat transmission and the coefficient of friction, and on the close approximation with which the empirical formula of Blasius, interpolates, for the coefficient of friction to which Latzko's formula corresponds, the experimental results even to the highest Reynolds Numbers. On the other hand, in a treatise now in preparation, which I expect to publish later and in which I have sought to incorporate the latest theories on turbulence, results are obtained which differ but little from the formulas of Blasius and Latzko, although expressed in a much more complex form. This confirms the approximate validity of Latzko's formula even at high velocities. In any case it is known that the results do not differ substantially, whatever may be the coefficient of transmission.

We now return to the consideration of a real fluid in which the frictional heat is no longer negligible and the temperature field must be regarded as a superposition of the two previously mentioned fields.

On designating the values of T_1 at the wall and at the outer surface of the boundary layer by T_{1p} and $T_{1\delta}$, we obtain

$$T_1 + \frac{u^2}{2g E c_p} = T_{1p} = T_{1\delta} + \frac{U^2}{2g E c_p}$$

and therefore

$$T_1 - T_{1\delta} = \frac{u^2}{2g E c_p}; \quad T_{1p} - T_{1\delta} = \frac{U^2}{2g E c_p}$$

Moreover, the effective temperature being $T = T_1 + T_2$, we must have $T_{1p} + T_{2p} = T_p$ at the wall and $T_{1\delta} + T_{2\delta} = T_0$ outside the boundary layer, if T_p is the temperature of the wall and T_0 is that of the undisturbed fluid. From these relations, on taking equation (4) into account, we obtain

$$T_{2p} - T_{2\delta} = T_p - T_0 - (T_{1p} - T_{1\delta}) = T_p - T_0 - \frac{U^2}{2g E c_p};$$

$$T - T_0 = T_1 - T_{1\delta} + T_2 - T_{2\delta} = \frac{U^2 - u^2}{2g E c_p} + \left(T_p - T_0 - \frac{U^2}{2g E c_p} \right) \left(1 - \frac{u}{U} \right)$$

The last formula connects the temperature field with the velocity field. It permits, for each law of the distribution of the latter, ~~in order to determine~~ the distribution of the former. On denoting by $\theta = T - T_0$ the temperature excess above the outside temperature and noting that

$\tau = \frac{U^2}{2g E c_p}$ is the temperature increase resulting from the adiabatic arrest of the fluid, we can write

$$\theta = \left(\theta_p + \tau \frac{u}{U} \right) \left(1 - \frac{u}{U} \right)$$

The effect of the frictional heat appears in the term $\tau \frac{u}{U}$ whose quadratic dependence on the velocity explains again why it is permissible to disregard it for low values. Naturally the preceding expression contains, as particular cases, the two from which is deduced by superposition, the one for $\tau = \theta_p$, the other for $\tau = 0$.

This demonstrates, moreover, the existence of a maxi-

maximum value of θ for $\frac{u}{U} = \frac{1}{2} \left(1 - \frac{\theta_p}{\tau}\right)$ which is ~~comprised~~ ^{reached} in the boundary layer and therefore exists in reality only for $-\tau < \theta_p < \tau$. In the cases of practical importance (for which θ_p is positive) the maximum occurs at a very short distance from the wall, due to the rapid increase in the value of u near the latter.

Coming to the most important question, that of heat transmission connected with the field now under investigation, it is known that the flow of heat from a temperature field deduced from two others by superposition must, because of its linear dependence on the field, coincide with the sum of the flows corresponding to the field components. In our case, since no heat transmission corresponds to the first field, it will coincide with that due to the second field.

Thus, if Q indicates the effective heat taken from the plate per time unit, it is given by Latzko's formula, in which $T_{ap} - T_{\infty}$ is replaced by the value just found

$$\frac{Q}{l} = 0.0356 \gamma c_p U \left(\frac{v}{U l}\right)^{1/5} \left(\theta_p - \frac{U^2}{2g E c_p}\right).$$

It is possible to arrive at this formula directly, if, in the heat balance from which Latzko's formula is deduced, account is also taken of the heat of friction. In the present scheme, according to which the plate is fixed and the fluid flows past it, this corresponds exactly to the energy lost by the fluid contained in the boundary layer. The total energy balance, expressing the fact that the heat transmitted from without and that of friction must be found entirely in the boundary layer, is therefore written

$$Q + \int_0^{\delta} \rho \frac{U^2 - u^2}{2} dy = \int_0^{\delta} E \gamma c_p (T - T_0) dy.$$

If $T - T_0$ is replaced by the previously found value, we obtain

$$Q = \left(T_p - T_0 - \frac{U^2}{2g E c_p}\right) \int_0^{\delta} \gamma c_p u \left(1 - \frac{u}{U}\right) dy \quad (6)$$

which coincides with the previous formula, when the Karman velocity distribution is assumed and γ and c_p are kept constant.

In reality, if the assumed constancy for c_p is largely justified, the case is otherwise for the density, whose variation can be no longer disregarded when the temperature variations at different points in the fluid are large, as is shown by the great temperature difference between the plate and the fluid and by the high velocity. Putting $\gamma = \gamma_0 \frac{T_0}{T}$, or approximately

$$\frac{\gamma}{\gamma_0} = 1 - \frac{T - T_0}{T_0} + \left(\frac{T - T_0}{T_0} \right)^2 - \dots,$$

under the integral and substituting the value found for $T - T_0 = \theta$ and integrating in the case in which Karman's law is assumed for the velocity, we obtain a correction factor of

$$1 - \frac{1}{5} \frac{\theta_p}{T_0} - \frac{8}{55} \frac{\tau}{T_0} + \frac{3}{55} \frac{\theta_p^2}{T_0^2} + \frac{4}{55} \frac{\tau \theta_p}{T_0^2} + \frac{18}{715} \frac{\tau^2}{T_0^2} - \dots,$$

a function of the temperature of the plate and of the velocity, decreasing with the increase of both. It is obvious therefore not only that the heat transmitted is not proportional to the temperature difference θ_p , but also that (as shown by the correction factor) it does not depend on it linearly, because of the diminution in the heat capacity of the fluid in the boundary layer with increase of temperature.

The confirmation of the existence of such a factor is sought in experimentation, which, so far as I know, has not yet been developed in this direction. The only experimental investigation of my theory, in which it was sought to determine (though only as a secondary object) the dependence of the transmission coefficient of the temperature (reference 5), shows a behavior very similar to that defined by the correction factor now written, when, due to the inadequate experimental velocity, it is put at $\tau = 0$. In the further developments, however, we shall disregard this correction factor, whose application does not seem justified by the present uncertainty regarding the practical values of the coefficients of heat transmission.

Retaining, therefore, Latzko's simple formula, modified, as has been shown, by the substitution of $\theta_p - \tau$ for θ_p , it is known, however, that this result is valid for any effective value of the coefficient of transmission, as shown by the simple consideration of equation (6). Moreover, it is permissible to assume that it is also val-

id for any form of the body considered, even differing from the laminar.

In order to prove this assertion, we start with the consideration of the equations for the boundary layer in the more general case where the pressure varies along the body, confining our researches to planar motion, though the results can be extended to motion in three dimensions. The hydrodynamic equations substituted for equations (1) are

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{dp}{dx} + \frac{1}{\rho} \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) \quad (1')$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

ρ being a function of p and T , or, in the present case, of both the variables x and y .

The limiting conditions are the same as for equations (1), but it is noted that here dx is taken at every point of the surface of the body according to the tangent to the surface in the plane of motion, and dy according to the normal.

As to the heat equation corresponding to equation (2), it may be obtained by applying to an element dV of the fluid the first principle of thermodynamics, $dQ = di - Avdp$. With respect to the element of volume and to the unit of time, it may be written

$$\gamma dV \left(\frac{di}{dt} - Av \frac{dp}{dt} \right) = \gamma dV \frac{dQ}{dt}$$

The heat, in part conducted into the element and in part produced by friction in the element itself is given, as for equation (2), by

$$\gamma dV \frac{dQ}{dt} = dV \left[\frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \frac{\mu}{E} \left(\frac{\partial u}{\partial y} \right)^2 \right]$$

Moreover,

$$\frac{di}{dt} = c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right), \quad \frac{dp}{dt} = u \frac{dp}{dx}, \quad Av = \frac{1}{E\gamma}$$

and the thermodynamic equation, divided by $\gamma c_p dV$,

becomes

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{u}{E c_p} \frac{1}{\gamma} \frac{dp}{dx} + \frac{1}{\gamma} \frac{\partial}{\partial y} \left(\frac{\lambda}{c_p} \frac{\partial T}{\partial y} \right) + \frac{\mu}{\gamma E c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (2')$$

with

$$\frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[\frac{\lambda}{c_p} \frac{\partial T}{\partial y} \right] = 0 \quad \text{for } y = \delta.$$

For the turbulent boundary layer, utilizing, as before, $\frac{\lambda_{\text{turb}}}{E c_p \mu_{\text{turb}}} = 1$ and combining equation (2') with the first of equations (1') multiplied by $\frac{u}{E c_p}$ the terms containing $\frac{dp}{dx}$ are eliminated and the same equation (4) is obtained in the beginning, apart from the presence of the factor $1/\rho$ and the substitution of μ_{turb} for ν_{turb} in the second member. The first of the solutions is then

$$T_1 + \frac{u^2}{2g E c_p} = \text{const.},$$

which now includes the whole effect of the heat of friction. On putting, as at first, $T - T_1 = T_2$, the equation in T_2 , identical with the one obtained from equation (2'), by canceling the last term, is not homogeneous, thus differing from the preceding case. It is easy, however, to make it homogeneous by noting that, outside of the boundary layer, the equation in T_2 yields

$$U \frac{dT_{2\delta}}{dx} = \frac{U}{E c_p} \frac{1}{\gamma} \frac{dp}{dx}, \quad \text{and therefore} \quad \frac{u}{E c_p} \frac{1}{\gamma} \frac{dp}{dx} = u \frac{dT_{2\delta}}{dx}.$$

By substituting in the equation in T_2 and noting that $T_{2\delta}$ is a function ~~of the sole~~ ^{only of} x , we obtain the homogeneous equation

$$u \frac{\partial(T_2 - T_{2\delta})}{\partial x} + v \frac{\partial(T_2 - T_{2\delta})}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left[\mu_{\text{turb}} \frac{\partial(T_2 - T_{2\delta})}{\partial y} \right],$$

of a form analogous to that in T_2 in the case of the flat plate, but no longer satisfied by a law of distribution strictly similar to that of the velocity, since equation (1') in u is no longer homogeneous nor analogous to the preceding equation. In practice it is also customary to assume for the velocity a distribution similar to that valid

for the plate. In the same order of approximation, the law of similitude could also be assumed to be valid. It is possible, however, to arrive at the same result without resorting to this assumption. In fact, due to the homogeneity of the last equation and the constancy of $T_{2p} - T_{2\delta}$, the solution of the last equation may be put in the form

$$\frac{T_2 - T_{2\delta}}{T_{2p} - T_{2\delta}} = f(x, y, U).$$

For the flat plate we have, as already seen, $f(x, y, U) = 1 - \frac{U}{U}$. On utilizing this relation in equation (5), noting that $u = u(x, y, U)$, $\delta = \delta(x, U)$ and putting

$$\int_0^\delta \gamma c_p u f(y, x, U) dy = \alpha(x, U)$$

(the coefficient of transmission when no account is taken of the heat of friction), the heat removed from the body up to the point of the abscissa (curvilinear) x is given by

$$Q = \alpha(x, U) (T_{2p} - T_{2\delta}).$$

Analogously with the foregoing case, since $T_{1p} - T_{1\delta} = \frac{U_\delta^2}{2g E c_p}$, U_δ being the local velocity outside of the boundary layer, we have

$$T_{2p} - T_{2\delta} = (T_p - T_{1p}) - (T_\delta - T_{1\delta}) = T_p - T_\delta - \frac{U_\delta^2}{2g E c_p}.$$

Since, in the hypothesis of adiabatic transformations

$$T_\delta + \frac{U_\delta^2}{2g E c_p} = T_0 + \frac{U^2}{2g E c_p}, \quad T_0 \text{ and } U \text{ indicating the temperature and velocity of the undisturbed current, we obtain definitely}$$

$$Q = \alpha(x, U) \left(T_p - T_0 - \frac{U^2}{2g E c_p} \right) = \alpha(x, U) (\theta_p - \tau).$$

Consequently we may express the following law.

The transference of heat between any object and a fluid flowing by it, when it is no longer possible to disregard the heat developed by friction, is governed by the same law that applied when it is negligible, provided the

temperature to which the fluid is brought by adiabatic arrest is considered as its temperature.

The result (although it can be formulated so simply) has important consequences. In order to explain it briefly, it is convenient to refer to the case of the flat plate just considered. It is immediately obvious that the transmission of heat, which is zero at zero velocity, is also nullified for $\frac{U^2}{2g \gamma c_p} = \theta_p$. This statement contains nothing new since, for $\theta_p = \tau$, we return to the known case of a heat-proof wall.

From the nullification of Q for two values of the velocity is deduced the presence of an intermediate maximum which is easily defined by a simple derivation with respect to U , and θ_p assumed to be constant. It is then found that for $\tau = \frac{2}{7} \theta_p$ and therefore for a velocity but little above half that for which Q is nullified, Q assumes the maximum value of

$$\frac{Q}{l} = 0.0356 \gamma c_p \left(\frac{v}{l} \right)^{1/5} \left(\frac{4g \gamma c_p}{7} \right)^{2/5} \frac{5}{7} \theta_p^{7/5}$$

Hence $\alpha = \frac{Q}{l \theta_p}$ passes through a value that is insuperable for every θ_p assigned and that increases slowly with θ_p .

Above the velocity which nullifies the transmission, this becomes negative, i.e., the heat, instead of passing from the plate to the fluid, passes in the opposite direction and, as the velocity increases, the coefficient of transmission increases indefinitely in the negative direction.

In order to illustrate more clearly the behavior thus defined, there are plotted in Figure 1, in terms of the velocity and in correspondence with certain values of θ_p , the curves for the quantities of heat carried off by each square meter of a plate one meter deep. In the calculation there are assumed for γ , c_p , and v the values relative to the air at a temperature of 20°C (68°F). Since it is practically possible to assume that for a thin wing the behavior is nearly the same as that of the flat plate, the graph gives an idea of the possibilities of a wing radiator at zero altitude. A mean temperature of the radiator

of 80°C (176°F.) being assumed, the corresponding curve is the one for $\theta_p = 60^{\circ}$. This shows immediately how the cooling power of a water radiator passes through a maximum value at 200 m (656 ft.) per second and decreases beyond this velocity to become zero at the velocity of sound. Above 350 m (1,148 ft.) per second the heat transmission becomes negative, the cooling power is converted into heating power, and the radiator becomes harmful. In order to increase its range of applicability, it would appear possible to use some liquid having a higher boiling point than water, as has already been done in practice, though for other reasons. With ethylene glycol ("Prestone"), for example, θ_p could reach 120°C (248°F.) and the cooling power would remain up to 500 m (1,640 ft.) per second. The conversion into heating power would still take place, however, above a certain velocity. The radiator would then become useless, even before the cooling power is nullified, since, as the latter approaches zero, the area requisite for giving off a certain quantity of heat would increase toward infinity.

Matters would be still worse if the results of this investigation were applied to high-speed flight at high altitudes (stratosphere) in what is termed "superaviation." (Reference 6.) In this case, for a given airplane and a given take-off speed, there would be, for every different flight speed, an economic altitude at which the efficiency would be the greatest. In the heat-transmission formula it would be necessary to take into account both the varying air density and the temperature drop at the different altitudes and consequently at the different speeds.

This was done for the two cases, radiator with water and with ethylene glycol, in the case (developed and calculated in the articles referred to) when the economic speed at zero altitude is 80 m (262 ft.) per second. The results are plotted in Figure 2. Evidently the portions of the curve running from 0 to 80 m (262 ft.) per second coincide with the corresponding curves of Figure 1. Beyond these speeds the airplane gains altitude and both the air density and temperature begin to decrease with opposite effects on the transmission of heat. While the lower density is unfavorable, the lower temperature of the air (assuming the temperature of the radiator to remain the same) facilitates the transfer of heat, especially in the case of the water radiator for which the relative increase of θ_p is much more pronounced than for the other. Hence,

while the cooling power decreases for the latter and the altitude shows hardly any increase, the decrease in density prevailing over the increase in θ_p , for the former it continues to increase until, as the stratosphere is approached and the temperature gradient decreases, the diminution of the density gains the advantage.

In both cases the maximum cooling power is reached below 100 m (328 ft.) per second, above which it falls rapidly, so that, as it is nullified for the same values of the speed as at zero altitude, the field suitable for the use of the radiator is greatly restricted in both cases.

This fact is still more clearly shown by the curve S/Q described below the first curves in Figure 2 and inverted with respect to them. It is known that the area required to transmit a given quantity of heat increases rapidly after the minimum is reached at 100 m/s, so that, at the velocity of sound, an area equal to four times the minimum is required in both cases.

It should be noted that any increase in the radiating surface beyond the total area of the wings and fuselage of the airplane would, in any case, be detrimental for the efficiency, by increasing the drag which is already so pronounced at high speeds.

Lastly, it is well to call attention to another question which, without being so vital as the cooling of the engine, merits some consideration, namely, that of the very high temperature excess on the surface and inside the cabin (G. A. Crocco, loc. cit.). Evidently, if the wall were heatproof, and consequently $\theta_p = \tau$, its temperature would exceed the outside value of τ and would reach values of several hundred degrees. Due to the poor conductivity of the wall, the temperature in the cabin would not be very seriously affected, but the mechanical resistance of the material of which the wall is composed would be diminished.

If, however, as is actually the case, the heat can pass through the wall and hence be transmitted by the boundary layer, the formula found for the transmission makes it possible to determine for any velocity the temperature excess θ_p of the wall in terms of the heat transmitted through it. Conversely, if it is desired that the temperature of the wall should not exceed a certain value, the quantity of heat to be carried off can be de-

terminated from the temperature excess.

This was done in Figure 3 in the case of superaviation when the bond between altitude and velocity was the same as that used shortly before. Here it is also assumed that the behavior of a streamline body is similar to that of the plate which, for example, is assumed to have a length of one meter. For any other length of l meters it is necessary to divide the values of Q/S by $l^{0.2}$.

The more logical and convenient solution is to cover the outside of the resisting metal wall with an insulating and refractory wall, internally cooled, for example, by liquid air. In this case there might be a great temperature excess of the outer refractory wall, while the small amount of heat transmitted would be absorbed by the cooling medium, without the metal wall or the interior of the cabin being detrimentally affected by the great temperature excess. The consumption of the cooling liquid would be very small.

This problem could be solved without very great difficulty as compared with the preceding problem of the radiator for which the high velocities seemed to present an insuperable obstacle. This confirms the assertion (G. A. Crocco, loc. cit.) that, at least as regards cooling systems differing from those now in use, the solution of the problem of extreme velocities (sonic and supersonic), toward which the trend now is, cannot be found by way of the explosion engine.

Translation by Dwight M. Miner,
National Advisory Committee
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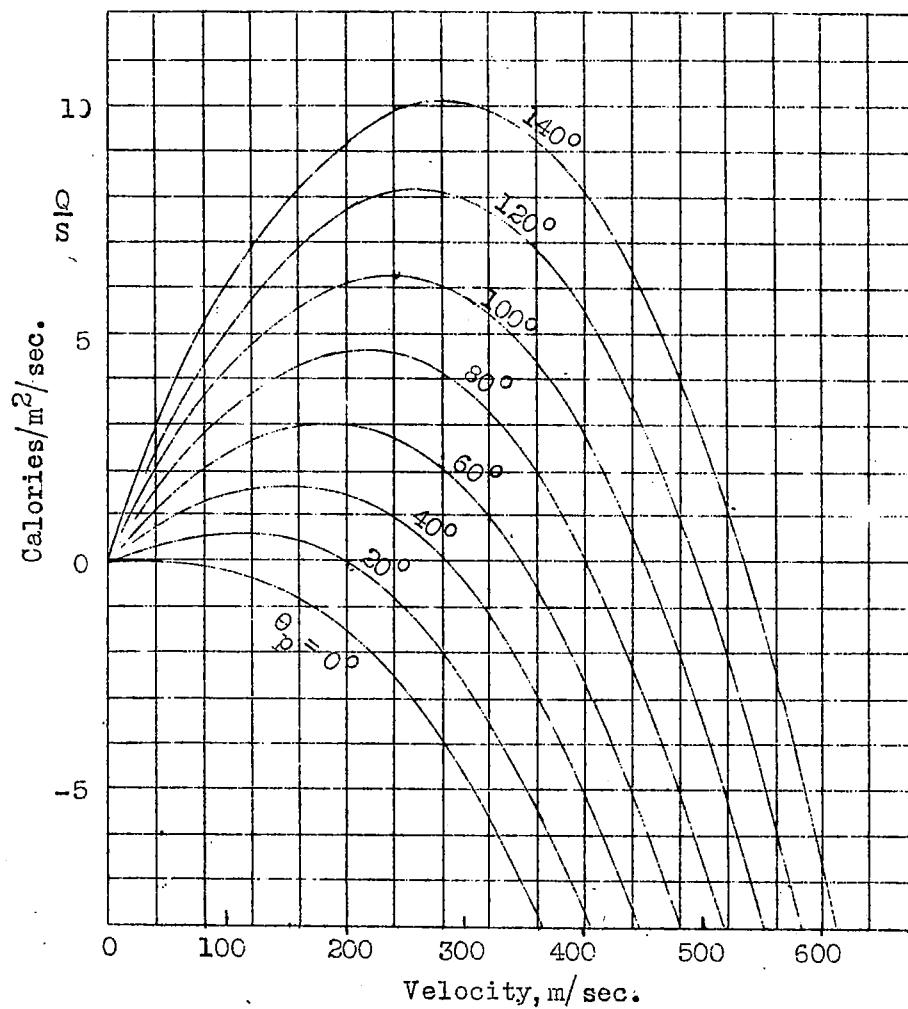


Fig. 1

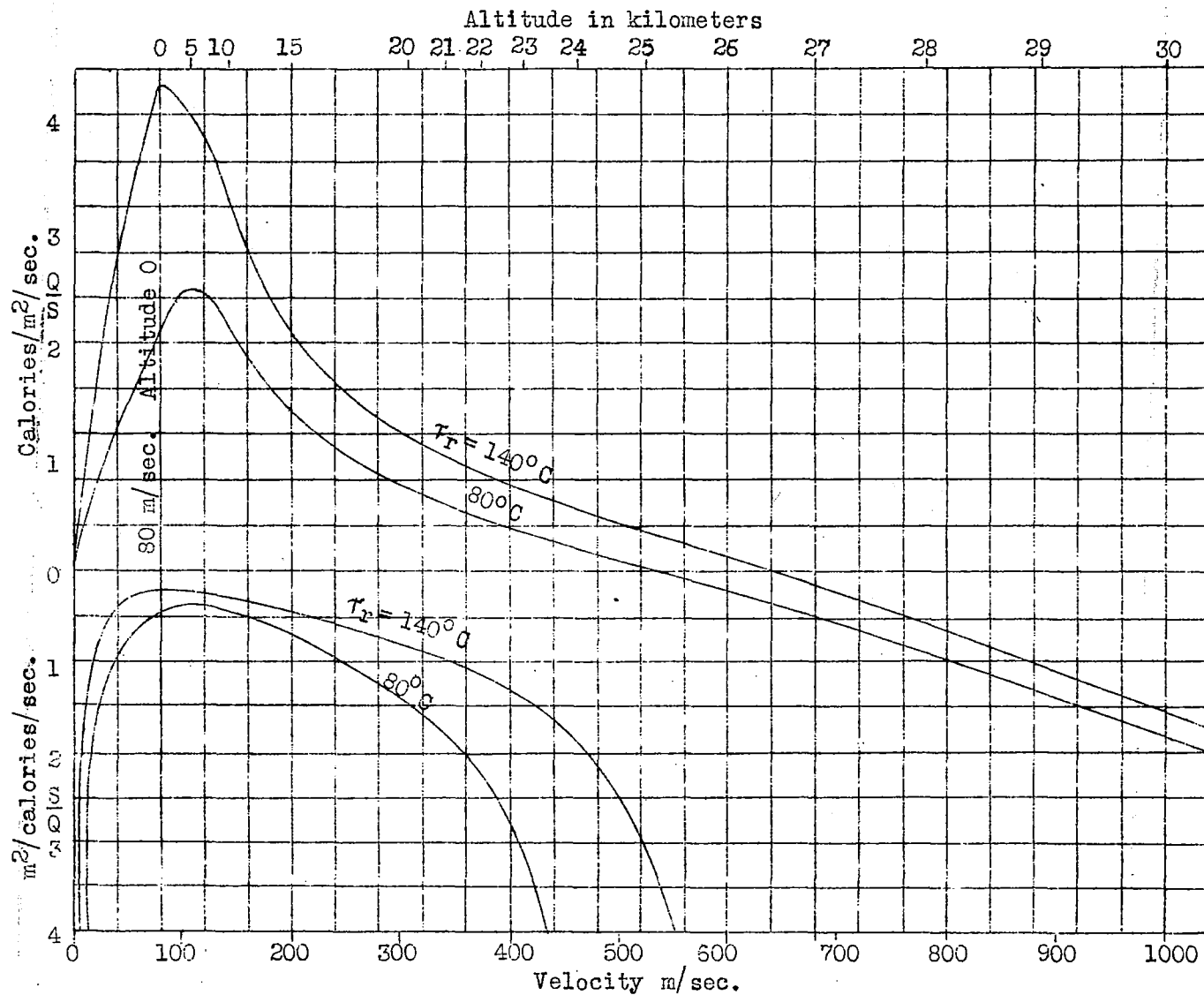


Fig. 2

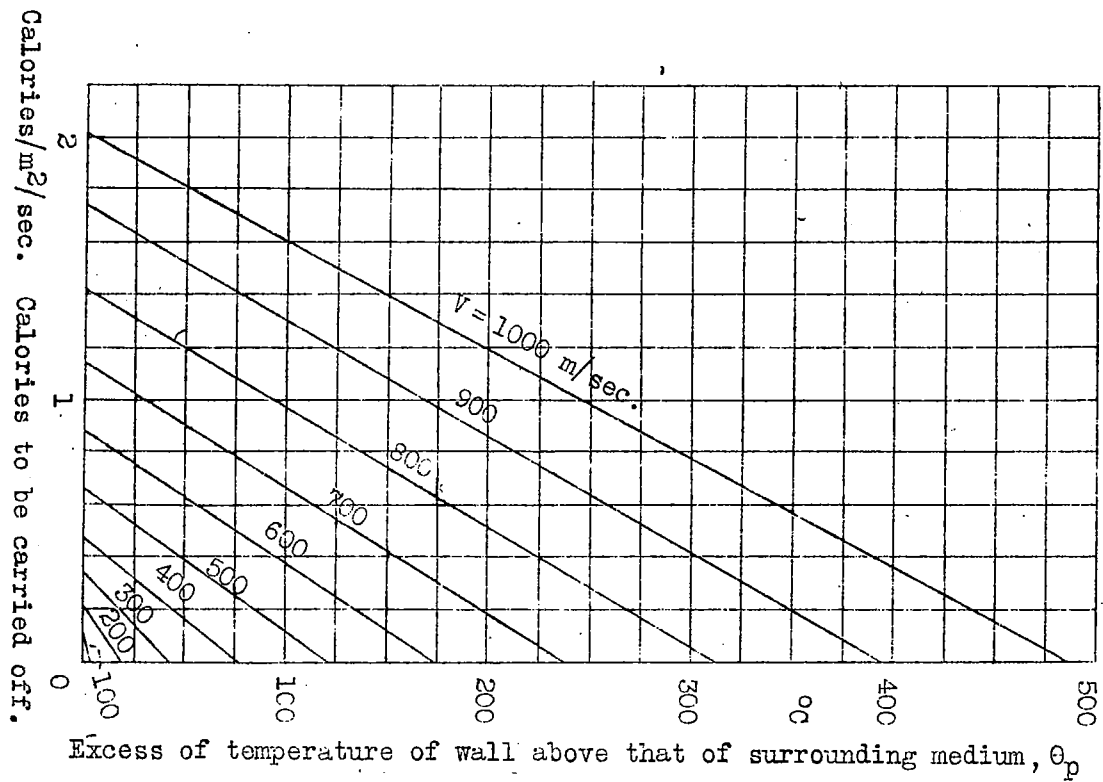


Fig. 3

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